The Fourth Essay on Methodological Background in Finance
— The Basic Model of Investment and Financial Decisions of the Corporation —

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4.1 Introduction and Notation

This essay gives a basic model of the financial behaviour of corporations. The fundamental financial equation derived in Section 4.3 is to serve as a starting point of the analytical framework for and the empirical study of the *dynamic* financial behaviour of the corporation in the subsequent essays. The effects of relevant financial variables on the capital structure ratio in a *static* equilibrium context are briefly examined in Section 4.2. An example of the dynamic adjustment process based upon the fundamental financial equation is presented in Section 4.4. The final section introduces the investment decision function to the basic model. The main purpose of this essay is to draw a sketch or to write a scenario of the basic model for the empirical study, and not to develop a rigorous mathematical model. Elaboration and extension of the basic model are made in The Fifth Essay.

Several characteristics and assumptions of the framework developed below are as follows. The fundamental relationship between, e.g., capital structure, dividend policy, and the investment decision is analysed for the representative corporation. It is assumed that the representative corporation is listed on a securities market and that one can freely interchange the concept of the individual and the aggregate, e. g. , a manufacturing corporation and its manufacturing industry. Relevant decisions for the corporation other than the financial decision, such as marketing, production, technology, advertising, selling, etc. , are not explicitly taken account of in the framework. Instead of considering the whole set of decisions, it is assumed that some of these decisions are evaluated in the markets and duly reflected in the financial results of the corporation as described in its financial statements (e. g. , the balance sheet and the income statement). The markets may not be perfect. The major characteristic of markets is their process of evaluation and their feedback machanism.[1]

The framework is somewhat different from the orthodox approach in finance. By "orthodox", I mean the theory of finance deriving from Modigliani and Miller in 1950s. Their approach can be characterised as a paradigm or a research programme based upon the neoclassical hard core.[2] The analytical framework in this essay is much more fundamental and flexible than the neoclassical one. First, in the orthodox theory, the investment function does not have any significant role or is assumed as independent of financing decisions. In Section 4.5, a simplified investment function is presented. Secondly, the neoclassical theory of the firm pays no attention to financial constraints at all. It is assumed, for example, that all the desired finance for growth can be immediately supplied through the notional markets. The framework in this essay may be extended to consider non-equilibrium phenomena in the markets and the two-stage decision making process of the corporation. In the first stage, the corporation behaves as if there were no financial constraints. The notional investment func-

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1 See Section 3.3 of The Third Essay.

2 See Section 3.2 of The Third Essay.

tion can be derived in this stage. However, the corporation soon meets the actual problems such as unfilled demand for growth, inflation, the growth of the economy, scarcity of resources, etc. In the second stage, the corporation is constrained by some of these actual problems.[3] Thirdly, the orthodox theory assumes that "uncertainty" can be directly reduced to calculable "risk". Instead of an explicit consideration of uncertainty or risk, I assume a simple adaptive process for forming expectations.[4] Finally, the fundamental approach may be easily extended to cover other dynamic disequilibrium aspects of the corporation and the markets, and to cover investment-financing decisions simultaneously in the empirical context. In brief, the approach might be called a Keynesian foundation for the theory of finance.[5]

Only the first-stage of the decision model is presented below.[6] The notation for the basic model is as follows:

K = total assets
E = shareholders' equity
B = total debts $(=K-E)$
S = capital stock
R = earnings retained $(=E-S)$
P = net income
D = dividends
$\hat{R} \equiv dR/dt$ = addition to retained earnings $(=P-D)$
c = the capital structure ratio $(=E/K)$
c^* = the dynamic equilibrium capital structure ratio
c^m = the static equilibrium capital structure ratio
r = profitability (= net operating income/K)
r^e = expected profitability
h = the rate of interest
x = the rate of taxes
e = the return on equity $(=P/E)$
y = the dividend payout $(=D/P)$
v = the dividend rate $(=D/S)$
s = the capital stock ratio $(=S/E)$
g = the rate of growth of total assets $(=\dot{K}/K)$
δ = the dividend ratio over equity $(=D/E)$
n = the new share issuing ratio $(=\dot{S}/\dot{K})$
t = time

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[3] See The Fifth Essay.

[4] For a succinct discussion of uncertainty, risk, and adaptive expectations, see Barro and Grossman [1976, pp.4–5].

[5] For more discussions, see Section 3.5 of The Third Essay.

[6] Empirical analyses for the first-stage of the decision model are given later.

4.2 A Static Equilibrium

To investigate the effects of relevant financial variables on the capital structure ratio of the corporation in a static cotext, it might be instructive to review briefly the well-known Proposition of Modigliani and Miller. Under several assumptions,[7] the following static equilibrium equation can be obtained as a result of arbitrage :

$$c^m = \frac{(r-h)(1-x)}{e-h(1-x)} \qquad (4.2.1).$$

Note that all variables in Eq. (4.2.1) are expressed in market values ; [8] that Eq. (4.2.1) is *not* an identity but shows an equilibrium relation ; and that e and r are treated as random variables by Modigliani and Miller.

The effect of each variable on the equilibrium capital structure ratio is as follows :

$$\frac{\partial c^m}{\partial r} > 0$$

$$\frac{\partial c^m}{\partial h} = -\frac{(1-x)[e-r(1-x)]}{[e-r(1-x)]^2} < 0^9$$

$$\frac{\partial c^m}{\partial x} = -\frac{e(r-h)}{[e-h(1-x)]^2} < 0$$

$$\frac{\partial c^m}{\partial e} = -\frac{(r-h)(1-x)}{[e-h(1-x)]^2} < 0.$$

The effects of profitability (r), the rate of interest (h), the rate of taxes (x), and the return on equity (e) on the equilibrium capital structure (=financial stability) are familiar and need no explanations.

The fatal weakness of the static model is that it fails to include explicitly the rate of growth and the investment decision function. Further, the effect of dividend policy on the capital structure cannot be analysed in the above formulation. The subsequent sections attempt to take account of these problems in a dynamic context.

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7 Modigliani and Miller [1958] and [1963].

8 For discussions of ex-post management decisions based on market values and ex-ante decisions on book values, see Vickers [1968], Turnovsky [1970] and Herendeen [1974]. Under other simplified assumptions, one can derive Eq. (4.2.1) in book values. See the next section.

9 $e=r(1-x)(r-h+ch)/cr>r(1-x)$, since $cr-(r-h+ch)=(r-h)(c-1)<0$, assuming $r>h$ and $cm<1$.

4.3 The Fundamental Financial Equation

The fundamental equation for the dynamic financial behaviour of the corporation is derived in this section. Assume that net operating income (rK) is equal to the sum of interest charges (hB), dividend payments (D), taxes $([rK-hB]x)$, and addition to retained earnings $(\dot{R}\equiv dR/dt)$:

$$rK=hB+D+(rK-hB)x+\dot{R} \qquad (4.3.1).$$

Since $B=K-E$, $D=\delta E$, and $R=E-S$, Eq. (4.3.1) can be written as:

$$rK=h(K-E)+\delta E+[rK-h(K-E)]x+\dot{E}-\dot{S}.$$

Divide both sides of the above equation by K:

$$r=h(1-c)+\delta c+[r-h(1-c)]x+\frac{\dot{K}}{E}-\frac{\dot{S}}{E}.$$

But since $\dot{E}/K=\dot{c}+gc$[10], and $\dot{S}/K=ng$, one can obtain the following nonhomogeneous differential equation for the capital structure ratio of the corporation:

$$\dot{c}=[h(1-x)-(g+\delta)]c+ng+(r-h)(1-x) \qquad (4.3.2).$$

Setting $\dot{c}=0$, the equilibrium capital structure ratio (c^*) is expressed as:

$$c^*=\frac{ng+(r-h)(1-x)}{g+\delta-h(1-x)} \qquad (4.3.3).$$

Compare Eq. (4.3.3) with Eq. (4.2.1). The equilibrium capital structure in the last section (c^m) is a special case in the sense that the rate of growth of total assets (g) is assumed to be zero and all the earnings are distributed to the shareholders $(e=\delta)$. The dynamic equilibrium capital structure (c^*) can be explicitly explained by the rate of growth, the new share issuing and a dividend policy variable as well as by the other financial variables.

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10 $\dot{c}\equiv dc/dt=d(E/K)/dt=(K\dot{E}-\dot{K}E)/K^2=\dot{E}/K-gc.$

Some comparative static results are obtained :

$$\frac{\partial c^*}{\partial r} > 0$$

$$\frac{\partial c^*}{\partial h} = \frac{(1-x)[ng+r(1-x)-(g-\delta)]}{[g+\delta-h(1-x)]^2} < 0^{11}$$

$$\frac{\partial c^*}{\partial \delta} < 0$$

$$\frac{\partial c^*}{\partial n} > 0$$

$$\frac{\partial c^*}{\partial g} = \frac{n[\delta-h(1-x)]-(r-h)(r-x)}{[g+\delta-h(1-x)]^2} \lesseqgtr 0.$$

The effects of profitability (r) and the rate of interest (h) on the equilibrium capital structure are the same as obtained in the last section. The effects of the dividend payments (δ) and the new share issuing ratio (n) are familiar.

The growth effect on the capital structure can be positive, negative, or irrelevant.[12]

Eq. (4.3.2) is a non-homogeneous differential equation, for which the general solution is given as follows :

$$c(t) = [c(0)x + \frac{B}{A}] \exp(At) - \frac{B}{A},$$

where $A = h(1-x) - (g+\delta)$,

and $B = ng + (r-h)(1-x)$ (4.3.4).

Eq. (4.3.4) is called a "fundamental dynamic equation for the financial behaviour of corporations". By definition of A and B, the time path of the capital structure ratio is determined by the initial condition $[c(0)]$, profitability (r), the rate of interest (h), the rate of growth (g), the rate of taxes (x), the new share issuing ratio (n), and the dividend ratio (δ).

Thus far, the dividend ratio over shareholders' equity (δ) has been used as a dividend policy variable. The fundamental financial equation (4.3.4) can be easily modified for the dividend payout (y), because the relationship $\delta = msy$ holds, where m is the earnings per share (P/S) and s is the capital stock ratio (S/E). Assume for simplicity that m and s are kept constant over time. Then, the fundamental equation for the dividend payout can be written as a function of r, g, h, and y : [13]

$$c = c(r, g, h, y) \qquad (4.3.5).$$

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[11] $ng + r(1-x) < g + \delta$, since $c^* < 1$.

[12] See footnote 17 below.

[13] x is also assumed constant over time.

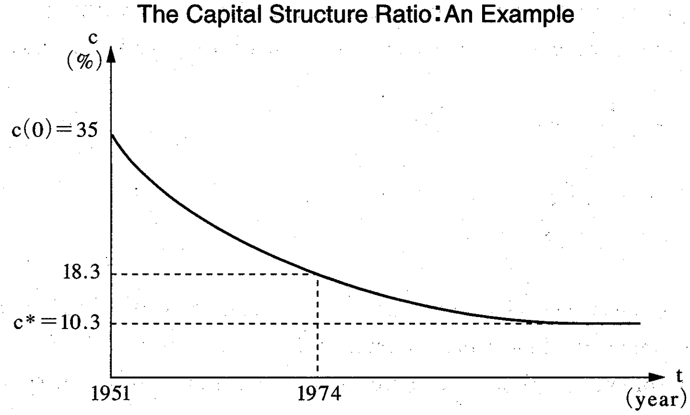
Another expression is possible for the dividend rate $(v \equiv D/S)$[14], assuming s is constant over time :

$$c = c(r, g, h, v) \qquad (4.3.6).$$

One can examine the dynamic time path of the capital structure more specifically. Assume for the time being that explanatory financial variables are constant over time. Then the capital structure ratio is monotonically declining or rising depending on the sign of A, for $dc/dt = A[c(t) - c^*]$ holds from Eqs. (4.3.2), (4.3.3) and (4.3.4). In order that the equilibrium capital structure is lower than one, $ng + r(1-x) < g + \delta$ must hold. Therefore, the sign of A is negative, assuming $h < r$. The time path of the capital structure ratio is monotonically converging to the equilibrium from above if the initial value is higher than the equilibrium and converging from below if the initial value is lower than the equilibrium.

Fugure 4.1 is an example where $r = 10\%$, $G = 15\%$, $h = 8\%$, $\delta = 6\%$, $x = 50\%$, $n = 50\%$, and therefore, $A = -0.17$ and $B = 0.0175$. These values approximately correspond to those for Japanese Manufacturing Industry.[15] In Figure 4.1, the initial value of capital structure is 35%, which was the value for Japanese Manufacturing Industry in 1951, and the equilibrium capital structure ratio is calculated as 10.3% from Eq. (4.3.3).

Figure 4.1
The Capital Structure Ratio : An Example



In sum, the fundamental financial behaviour of the corporation can be expressed as a function of other relevant financial variables as follows :

$$c = c(r, g, h, n, \delta)$$
or
$$c = c(r, g, h, y)$$[16]
or
$$c = c(r, g, h, v)$$

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[14] For more discussions, see Section 6.2 of The Sixth Essay.

[15] Cf. Figure 6.3 of The Sixth Essay.

[16] Assume x and s = constant.

with following comparative static properties : [17]

$$c^*_r>0,\quad c^*_g>0,\quad c^*_h>0,\quad c^*_\delta>0,$$

$$c^*_y>0,\quad c^*_v>0,\quad c^*_n>0,$$

Empirical studies will be based upon linearised fundamental equations, e. g. :

$$c=k_0+k_1r+k_2g+k_3h+k_4y \qquad (4.3.7).$$

4.4 A Dynamic Process

This section examines a dynamic relationship between capital structure and dividend policy. Assume that for the dividend payments per equity (δ=D/E) the following Lintner process is adopted : [18]

$$\begin{aligned}\dot{\delta}\equiv\frac{d\delta_t}{dt}&=\alpha[\delta_t-\delta^+]\\ &=G(\delta_t),\quad \alpha<0\end{aligned} \qquad (4.4.1).$$

δ^+ is called a target dividend ratio.

The time path of the capital structure ratio is specified by the fundamental financial equation

$$\begin{aligned}\dot{c}&=[h(1-x)-(g+\delta)]c+ng+(r-h)(1-x)\\ &=F(c_t,\ \delta_t)\end{aligned} \qquad (4.4.2).$$

The dynamic process can be analysed by the two differential equations (4.4.2) and (4.4.1). Let J (c,δ) be the Jacobian matrix of Eqs. (4.4.2) and (4.4.1) :

$$J(c,\ \delta)\equiv\begin{pmatrix}\dfrac{\partial F}{\partial c} & \dfrac{\partial F}{\partial \delta}\\[2ex] \dfrac{\partial G}{\partial c} & \dfrac{\partial G}{\partial \delta}\end{pmatrix} \qquad (4.4.3).$$

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17 c^*_r, for example, $=\partial c^*/\partial r$. For the dividend payout or the dividend rate equation, one can easily confirm $\partial c^*/\partial g<0$.

18 See Lintner [1956]. For empirical studies of the Lintner process for the dividend payout, see e.g., Fama [1974].

From Eq. (4.4.2) :

$$\frac{\partial F}{\partial c}=h(1-x)-(g+\delta),$$

assuming h,g, δ, n, r, and x, are independent of c, and

$$\frac{\partial F}{\partial c}=-(\frac{\partial g}{\partial \delta}c+\frac{\partial c}{\partial \delta}g+c+\frac{\partial c}{\partial \delta}\delta)+n\frac{\partial g}{\partial \delta},$$

assuming x, n, r, and h are independent of δ.
From Eq. (4.4.1) :

$$\frac{\partial G}{\partial c}=0, \quad \text{and}$$

$$\frac{\partial G}{\partial \delta}=\alpha.$$

If $J(c, \delta)$ has characteristic roots with negative real parts, i.e., if tr $J(c, \delta)<0$ and det $J(c, \delta)>0$, then the system of Eqs. (4.4.1) and (4.4.2) is stable.[19] From Eq. (4.4.3) :

$$\text{tr } J(c, \delta)=h(1-x)-(g+\delta)+\alpha$$

$$\det J(c, \delta)=[h(1-x)-(g+\delta)]\alpha.$$

However, from Eq. (4.3.3), $h(1-x)-(g+\delta)<0$. Therefore, if $\alpha<0$, then tr $J(c, \delta)<0$ and det $J(c, \delta)>0$, and the system is stable.

The dynamic behaviour of the capital structure ratio and the dividend ratio can be investigated more closely by drawing an appropriate phase deagram.[20]

Setting $\dot{c}=0$ and $\dot{\delta}=0$, the equilibrium capital structure c* and the equilibrium dividend ratio δ* are :

$$c^*=-\frac{ng+(r-h)(1-x)}{h(1-x)-(g+\delta^*)}$$

$$\delta^*=\delta^+.$$

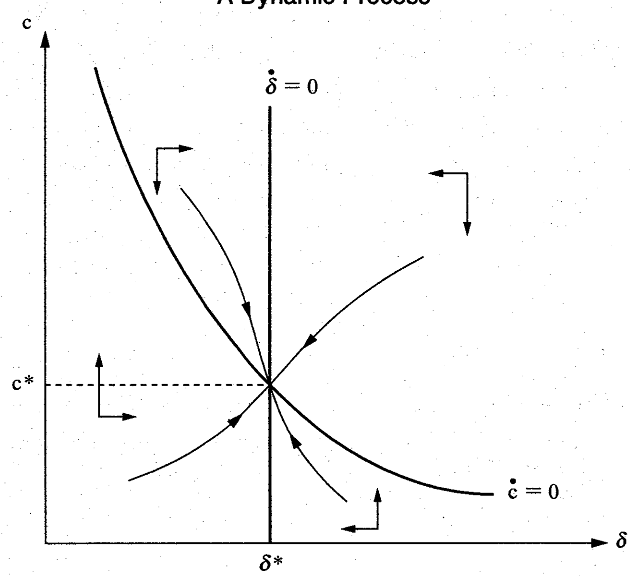
[19] See Olech [1963], Garcia [1972], and Samuelson [1947].

[20] For economic applications of the phase diagram technique, see, e. g., Wan [1971], Burmeister and Dobell [1970], Intriligator [1971], Shell [1969], Arrow [1968], and Dorfman [1969].

The following is a preliminary table for the phase diagram of ct and δt.

	$\delta < \delta^*$	$\delta = \delta^*$	$\delta > \delta^*$
$c < c^*$	$\dot{c} > 0$ $\dot{\delta} > 0$	$\dot{c} > 0$ $\dot{\delta} = 0$	$\dot{c} > 0$ $\dot{\delta} < 0$
$c = c^*$	$\dot{c} = 0$ $\dot{\delta} > 0$	$\dot{c} = 0$ $\dot{\delta} = 0$	$\dot{c} = 0$ $\dot{\delta} < 0$
$c > c^*$	$\dot{c} < 0$ $\dot{\delta} > 0$	$\dot{c} < 0$ $\dot{\delta} = 0$	$\dot{c} < 0$ $\dot{\delta} < 0$

Figure 4.2
A Dynamic Process



The local stability of the above dymanic system can be confirmed by the linearisation of the system based upon a Taylor expansion about an equilibrium point.[21] One can easily obtain the stable node for the above system.[22]

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[21] See Intriligator, op. cit., and Takayama[1974].

[22] See Boyce and Di Prima[1977] and Pontryagin[1962].

4.5 Investment Function and the Basic Model

The last section has studied very briefly the dynamic relationship between capitral structure and dividend policy. So far, the investment policy of the corporation has not been introduced explicitly. As is well-known, it has been pointed out that it is impossible to derive the investment function within a neoclassical framework.[23] Several alternative models are available.

One of the non-neoclassical investment functions is presented by Uzawa. Following the work of Penrose, [24] Uzawa derives the investment function in which the optimum rate of investment (I^*/K) is determined by the rate of discount (h) and the expected rate of profitability (r^e) : [25]

$$\frac{I^*}{K} = \psi(h, r^e), \quad \frac{\partial \psi}{\partial h} < 0 \text{ and } \frac{\partial \psi}{\partial r^e} > 0.$$

Another one is the animal spirits function : [26]

$$g = g(r^e), \quad g' > 0 \text{ and } g'' < 0.$$

One of the difficulties in the Uzawa model is that the definition of "real" capital K is somewhat ambiguous for empirical verification. In addition, it is based upon a choice-theoretic approach and the existence of certainty equivalents. It is not necessary to derive an investment function as a result of explicit maximising behaviour of the corporation. Rather, it should reflect active entrepreneurial elements or the animal sperits, on the one hand, and a trade-off between investment and relevant financial decisions, on the other. A simplified investment function which satisfies these basic requirements can be written as :

$$g = g(h, r^e, y)$$

$$\frac{\partial g}{\partial h} < 0, \quad \frac{\partial g}{\partial r^e} > 0, \quad \frac{\partial g}{\partial y} < 0, \qquad (4.5.1).$$

The active, creative and entrepreneurial element of the management is expressed in $\partial g/\partial r^e > 0$; costs of uncertainty are reflected in $\partial g/\partial h < 0$; [27] and the trade-off between the investment decision and the dividend policy is measured by $\partial g/\partial y < 0$.

The financial equation was :

$$c = c(r, h, g, y)$$

$$\frac{\partial c}{\partial r} > 0, \quad \frac{\partial c}{\partial h} < 0, \quad \frac{\partial c}{\partial g} < 0, \quad \frac{\partial c}{\partial y} < 0, \qquad (4.5.2).$$

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23 See Section 3.1 of The Third Essay.

24 Penrose [1959].

25 The objective of the corporation is to maximise the discounted cash flow. See Uzawa [1968] and [1969].

26 See Section 3.3 of The Third Essay.

27 The former is called "propulsive", and the latter "retroactive" factors by Kahn [1972].

The first stage consists of the investment decision function (4.5.1) and the fundamental financial equation (4.5.2).

It is possible to specify the dividend policy and expected profitability as reflecting some adjustment processes. If one assume that the dividend policy is expressed as an adjustment process of Lintner, then :

$$\dot{y} = \alpha (y - y^*)$$

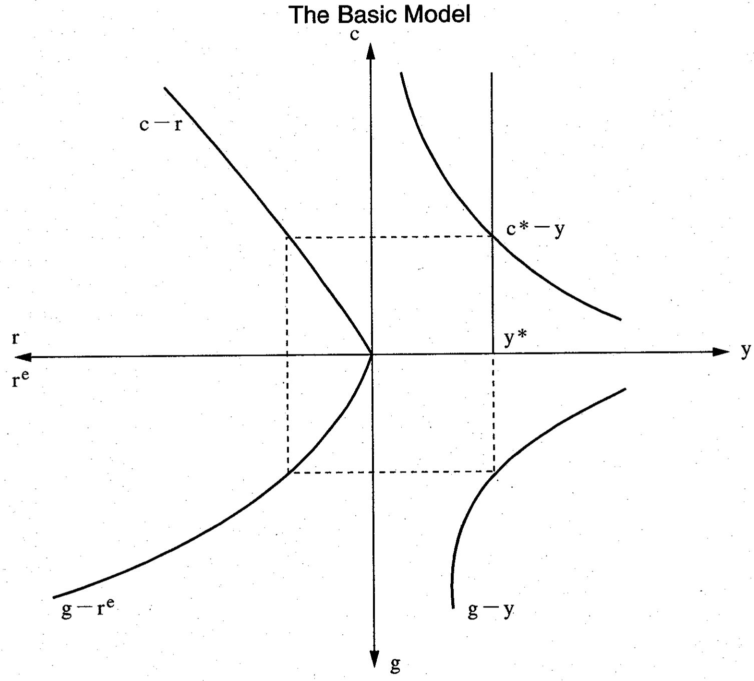
$$y^* = \text{ the target payout,} \quad \alpha < 0 \qquad (4.5.3).$$

The role of "expectations" may be very important for the specification of expected profitability. For simplicity, assume that expected profitability is adjusted as "adaptive expectations" :

$$\dot{r}^e = \beta (r - r^e), \quad \beta > 0 \qquad (4.5.4).$$

The system (4.5.1) ~ (4.5.4) is called the "basic model for the financial behaviour of corporations". Figure 4.3 illustrates the basic model.

Figure 4.3
The Basic Model



The capital structure-dividend policy relationship is depicted in the first quadrant. As analysed in Section 4.3, the equilibrium point (c*, y*) is stable assuming δ=msy. In the second quadrant, the linear relationship between capital structure and profitability is shown. The so-called animal spirits function is depicted in the third quadrant. The trade-off between dividend and investment is shown in the fourth quadrant.

The basic model is no more than exposition and recapitulation. A notable characteristic of the basic model is that the investment function is derived as if there is no financial constraint and the dynamic relationship between relevant financial variables is specified independently. This stage of deriving the notional investment function and the fundamental financial equation is called the first-stage decision. In the second stage, the investmint function is constrained by financial problems. Most of the dynamic models of corporations are inadequate in the sense that they have failed to treat the second-stage of the decision model.