# Memorization and Reasoning of Semantic Network Using PairedAssociate Mapping of Patterns 

Shoichi Suzuki ${ }^{\dagger}$


#### Abstract

In knowledge engineering, there is a semantic network as one method expressing knowledge. Inheritance of properties is possible for a semantic network, and it is a directed graph which consists of nodes and links. Memorizing a semantic network and reasoning there are usually performed by manipulating symbols.

A method of memorizing a semantic network and reasoning can be performed by pattern processing is presented here. In other words, it is shown that memory action and reasoning of a semantic network can be performed using model-construction operator $T$, similarity-measure function $S M$, and a paired-associate mapping $A$ suggested by S,Suzuki.

The five advantages are as follows : (1) The concept and the property memorized by adoption of pattern expression have been easily expressed by the linear-predictive-coefficient vector of the pronounced sound.. (2) The concept memorized and the property became strong to noise by adoption of a pattern model. (3) In two phases of memorizing and reasoning made by the association, it became strong to noise by adoption of the paired-associate mapping. (4) In the reasoning phase by association, it became strong to noise by adoption of the similaritymeasure function. (5) It can simply perform adding to a semantic network new triple that consists of concept 1 , property, and concept 2 by easy correction of the paired-associate mapping and the similaritymeasure function.


## Key words

semantic network pattern model

Reasoning processing by the patterns similarity-measure function
paired-associate mapping

## 1. Introduction

There are ( $1^{*}$ ) a procedural representation and (2*) a declarative expression for expressing knowledge in knowledge engineering. The expression by program language LISP is typical of the procedural representation and the expression by program language Prolog is one example of declarative expression. It divides roughly into the latter declarative expression, and there are four kinds [6]:
(1\#) Predicate calculus which adopted Prolog etc. as an expression language.
(2\#) Production rule.
(3\#) Semantic network.
(4\#) Frame.
These declarative expressions are usually realized as processing which manipulates a sequence of symbols [7].
Inheritance of propertiese is possible for a semantic network, and it is directed graph which consists of a set of nodes and links. This semantic network to which a node expresses a concept and the link expresses the relation between nodes is a collection of the triples, for example,
(2\$) <Taro IS-A human>, etc.
expressing the knowledge
(1\$)「Taro is a human」.
Taro, a human being, etc. express a concept and IS-A expresses the property, the attribute, the relation, etc. If a triple
(3\$) <human can thinking>
expressing the knowledge "human can think" is also memorized, the so-called " inheritance of property " is possible in a semantic network in the form of that
$(4 \$)<$ Taro can thinking> is reasoned from < Taro IS-A human> and <human can thinking>. If the recursive neural network of Elman (Elman, J.L.) that consists of four layers of
$(1 \%)$ an input layer $(2 \%)$ a hidden layer $(3 \%)$ an output layer $(4 \%)$ a primary memory layer which memorize the past activation situation only at the 1 unit time of the hidden layer provisionally, and have connection of feedback to the hidden layer are prepared is employed, it is clear that the grammar structure of complicated English containing a relative clause can be learned as an activation pattern (a distributed representation) [8], and a semantic network can be memorized. However, by this neural network, in order to attain the reasoning which can inherit an property attribute, another framework is needed.

In this paper, the semantic network which is a set of triples is memorized, and reasoning there is now realized as one associative processing of a pattern pair not different from symbol-sequence processing in which a symbol-sequence is manipulated.

In the semantic network of this paper, a triple <human IS-A animal> is memorized as a if-then-rule (1\&) if <human IS-A> then <human animal>.
When the element of a semantic network is for example,"<human IS-A animal>"and <human IS-A animal> is memorized as two pattern pairs <human IS-A> and <human animal>, reasoning expressed with modus ponendo ponens
(1) Fact: <human IS-A>> holds.
(2)Rule : It is <human animal> if it is <human IS-A>.
（3）Conclusion：＜human animal＞$\therefore$＜animal＞
about a declarative knowledge＜human IS－A animal＞is processed in the form of
＂pattern association of the pattern pair＜human animal＞is carried out from the pattern pair＜human IS－ A＞＂．
Five acquired advantages（1＠）～（5＠）brought about by four adopted fundamental design plans
I ．Human，IS－A，animal，etc．are expressed not as symbol sequences but as 16 －dimensional vectors of the real linear prediction coefficients of voice waveforms of the person who uttered these
II ．Furthermore，the patterns are prepared in the form where noise is removed from a voice waveform by asking for the model of a 32－dimensional linear prediction coefficient vector
III．Reasoning using a modus ponendo ponens is realized by work of association
IV．Memory similar to the obtained reasoning result is looked for by application of a method of maximum similarity which is as follows and is checked by the semantic network implemented in the Java language ：
（1＠）The concepts and the attributes memorized by adoption of pattern expression have been easily expressed by the linear prediction coefficient vectors of the pronounced sounds．
（2＠）The concept and the attribute which are memorized by adoption of a pattern model are strong to noise．
（3＠）A memory scene and a recall scene are strong to noise by adoption of the paired associate mapping．
（4＠）A recall scene is strong to noise by adoption of the similarity－measure function．
（5＠）When it must have to add new triples to an old semantic network，the old memorizing／reasoning function is maintained about this addition and this addition can be simply performed by easy correction of the paired－associate mapping and the similarity－measure function．

With this five advantages $(1 @) \sim(5 @)$ ，this research will offer the technology used as the foundation required to build a full－scale semantic network system．

Although the conventional semantic memory model is realized by sign sequence processing or the neural network［6］～［10］，this semantic memory model adopts pattern processing of a theory（SS theory［1］～［4］） suggested by S．Suzuki，is realized，and this realization becomes clear from three points（a），（b），（c）：
（a）In order to use instead of the pattern $\varphi$ the pattern model $T \varphi$ which can remove the noise which may be in a pattern $\varphi$ ，the model－construction operator $T$ of SS theory was used．
（b）The paired－associate operator $A$［5］by thinking over the paired－associate matrix［9］suggested by T．Kohonen in the $n$－dimensional Euclidean space $R^{n}$ to which the method of minimum square might be applied was obtained in general separable abstract Hilbert space making full use of the composition technique of SS theory ，and the operator $A$ was used as a paired－associational inferencer．
（c）In order to inspect whether the associative output from an associative inferencer has a semantic relation of being similar to one of the contents throats of memory，the similarity－measure function $S M$ of SS theory was used．

In addition，this function of memorization and reasoning on a semantic network is equipped with the performance beyond the function of memorization and reasoning of the conventional semantic network system done by symbol processing about noise－proof nature．It is because the three following character（1＠），（2＠）and （3＠）is materialized fundamentally in this system：
(1@) The modeling conclusion expression (6) of the model-construction operator $T$
(2@) The orthonormal expression (8) type of the similarity-measure function $S M$
(3@) The interpolation expression (A.11) of the paired-association operation $A$

## 2. Design method of semantic network system

The method of designing the semantic network system as one of the semantic memory models is briefly explained by this chapter.

### 2.1 Expression vector of a pair of symbol sequence

A semantic network can be expressed as set

$$
\begin{equation*}
<\mathrm{A}[\mathrm{j}] \mathrm{R}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>, j=1 \sim r \tag{1}
\end{equation*}
$$

of triples.
About symbols in which it is different from each other in the set

$$
\begin{equation*}
\mathrm{A}[\mathrm{j}], \mathrm{R}[\mathrm{j}], \mathrm{B}[\mathrm{j}], j=1 \sim r \tag{2}
\end{equation*}
$$

of symbol sequences, all are pronounced and we ask for the voice waveform. The voice waveform in the 1dimensional section $\{x \mid x=-p \sim+p\}$ of such a symbol sequence is set to be $f(x)$. The 1-dimensional section $\{x \mid x=-p \sim+p\}$ is mostly chosen as the middle portion in the utterance section. The linear prediction coefficient vector

$$
\vec{C}(f)=\operatorname{col}\left(C_{1}(f) \quad C_{2}(f) \quad \cdots \quad C_{q}(f)\right) \quad \text { (column vector) }
$$

of $f$ according to Appendix D is asked for. Here, since the orthogonal system of the expression (A.8) of Appendix B is obtained, inequality

$$
r \leq 2 q
$$

must be materialized. In this semantic network system,

$$
r=17, q=16
$$

were set up. Henceforth, $\vec{C}(f)$ may be simply written to be $\vec{C}$.
As opposed to the k-th triple

$$
\begin{equation*}
<A[j] R[j] B[j]> \tag{3}
\end{equation*}
$$

in a set of triples of expression (1), the「if condition then action」 type knowledge representation unit

$$
\begin{equation*}
\text { if }<A[j] R[j]>\text { then }<A[j] B[j]> \tag{4}
\end{equation*}
$$

is considered. Expression of the paired-sequence $\langle\mathrm{A}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ for a symbol showing a condition shall be a $2 q$ dimensional real vector, should connect the $q$-dimensional real value vector of $\mathrm{B}[\mathrm{j}]$ after the $q$-dimensional real value vector of $\mathrm{A}[\mathrm{j}]$, and shall have been obtained. We asks for the same $2 q$-dimensional real vector also from the sequence $<\mathrm{A}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ for a symbol showing an action.

### 2.2 The noise removable model $T \varphi$ of Pattern $\varphi$

It is considered that the expression vector of paired $j(=1 \sim r)$-th symbol sequence $<\mathrm{A}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ is an element(pattern) of $2 q$-dimensional Euclidean space $R^{2 q}$. The inner product $(\varphi, \eta)$ of $R^{2 q}$ and the norm $\|\varphi\|$ are defined by the expression (A.5). The $k(=1 \sim 2 q)$-th component $(\varphi)_{k}$ in this expression (A.5) is defined by the expression (A.3). It asks for the model(pattern model) $T \varphi_{j}$ of $\varphi_{j}$ according to 1 . of an
appendix．The adopted mapping $T$ which is called a model－construction operator is explained by its appendix 1.

Generally，there is the following two fundamental character in the pattern（pattern model）$T \varphi \in R^{2 q}$ called the noise removable model of the pattern $\varphi \in R^{2 q}$ of expression（A．1）：
（1\＃）（ $T$ absorbs multiplying by the any positive constant；cone property）

$$
\begin{equation*}
\forall \varphi, T(a \cdot \varphi)=T \varphi \text { for any positive real number } a \tag{5}
\end{equation*}
$$

（2\＃）（Idempotent property of $T$ ；conclusion of modeling）

$$
\begin{equation*}
\forall \varphi, T(T \varphi)=T \varphi \tag{6}
\end{equation*}
$$

Two above－mentioned matters（1\＃）and（2\＃）shows the following matters（1\＄）and（2\＄）respectively：
$(1 \$)$ The model of the pattern multiplied by positive arbitrary constants is the model of the original pattern which has not been multiplied by the constants．
（2\＄）The model of a model is the original model．
There is character in which small noise is removable from the pattern $\varphi$ of expression（A．1）in mapping $T$ because it can be understood from
（3\＃）（Noise removable nature）

$$
(T \varphi)_{k}=0 \text { if } e_{-}(k)<\frac{a_{k}}{\max _{\ell=1 \sim n}\left|a_{\ell}\right|}<e_{+}(k)
$$

being materialized．$(T \varphi)_{k}$ is the $k(=1 \sim 2 q)$－th component of expression（A．3）of B of expression $T \varphi$ here． Moreover，it is as follows if one pattern

$$
\eta=\operatorname{col}\left(\begin{array}{llllll}
b_{1} & b_{2} & \cdots & b_{k} & \cdots & b_{2 q}
\end{array}\right)
$$

is introduced and explained now ：
（4\＃）（Each model being equivalent which follows from removing deformation of patterns）
About all the $k(=1 \sim 2 q)$ ，if either in three conditions

$$
\text { (4\#_1) } e_{-}(k)<\frac{a_{k}}{\max _{\ell \in \neg n}\left|a_{\ell}\right|}<e_{+}(k) \wedge e_{-}(k)<\frac{b_{k}}{\max _{\ell=1 \sim n}\left|b_{\ell}\right|}<e_{+}(k)
$$

（4\＃＿2）$\frac{a_{k}}{\max _{\ell=1 \sim n}\left|a_{\ell}\right|} \leq e_{-}(k) \wedge \frac{b_{k}}{\max _{\ell=1 \sim n}\left|b_{\ell}\right|} \leq e_{-}(k)$
（4\＃＿3）$\frac{a_{k}}{\max _{\ell \in \neg-1}\left|a_{\ell}\right|} \geq e_{+}(k) \wedge \frac{b_{k}}{\max _{\ell=1 \sim n}\left|b_{\ell}\right|} \geq e_{+}(k)$
are materialized，the equivalent nature

$$
T \varphi=T \eta
$$

of two models will be realized．That is，it is shown that there is character to remove modification of a pattern in mapping $T$ ，that is to say，to extract character common to two patterns $\varphi$ and $\eta$ ．

When a set of triples of expression（1）is given，in a trial－and－error method，it is quite difficult to ask for patterns which corresponds to each symbol sequence of expression It has become clear by the method using the linear prediction coefficient of two paragraphs 2.1 and 2.2 for this to be possible，without applying a trial－and－ error method to almost．

## 2．3 Paired associate operator $A$

The paired－associate operator $A$ of expression（A．9）which recollects the model $T \eta_{j}$ of action pair symbol sequence $<\mathrm{A}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ is constructed from model $T \varphi_{j}$ of condition pair symbol sequence $<\mathrm{A}[\mathrm{j}] \mathrm{R}[\mathrm{j}]>$ about if－then－rule of expression（4）for any $j(=1 \sim r)$ according to appendix 2 ．This construction is a function of memory of a semantic network．

There is an interpolation property of expression（A．11）in the function of this memory．This interpolation property has guaranteed that the symbol sequence $<\mathrm{A}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ can be called correctly from the arbitrary symbol sequences $\langle\mathrm{A}[\mathrm{j}] \mathrm{R}[\mathrm{j}]>$ for any $j(=1 \sim r)$ ．

In addition，stage $A_{k+1}$ in the middle of construction is extension of $A_{k}$ ，and there is interpolation property of expression（A．12）also in stage $A_{k}$ in the middle of construction．

The addition of triple $<\mathrm{A}[\mathrm{r}+1] \mathrm{R}[\mathrm{r}+1] \mathrm{B}[\mathrm{r}+1]>$ with this new fact guarantees becoming easy．

## 2．4 Similarity－measure function $S M$

Each $\omega_{j}(j=1 \sim r)$ of expression（A．16）of Appendix C is set to be the $2 q$－dimensional real－number－ value vector $T \eta_{j}$ showing the action pair $<\mathrm{A}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ in triple $<\mathrm{A}[\mathrm{j}] \mathrm{R}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ of expression（3）．Namely

$$
\begin{equation*}
\omega_{j}=T \eta_{j}, j=1 \sim r \tag{7}
\end{equation*}
$$

will set up．
The similarity－measure function $S M\left(\varphi, \omega_{j}\right)$ which gives the grade to which pattern $\varphi$ resembles pattern $\omega_{j}$ is defined like expression（A．15）of appendix 3．．There is the three following properties（i），（ii ） and（iii）in the constructed similarity－measure function $S M$ ：
（i）（orthonormality）

$$
\begin{align*}
& \forall i, \forall j, S M\left(\omega_{i}, \omega_{j}\right)= \\
& \left\{\begin{array}{l}
1 \cdots i=j \text { のとき } \\
0 \cdots i \neq j \text { のとき }
\end{array}\right. \tag{8}
\end{align*}
$$

（ ii ）（probability condition，normalization）

$$
\begin{equation*}
\forall \varphi, \sum_{j=1}^{r} S M\left(\varphi, \omega_{j}\right)=1 . \tag{9}
\end{equation*}
$$

（iii）（invariance under mapping $T$ ）

$$
\begin{equation*}
\forall \varphi, \forall j \in\{1,2, \cdots, r\}, S M\left(T \varphi, \omega_{j}\right)=S M\left(\varphi, \omega_{j}\right) . \tag{10}
\end{equation*}
$$

The nomalized inner product $\operatorname{nip}\left(T \varphi, T \omega_{j}\right)$ of the form of expression（A．14）is not adopted as a degree of similar between two patterns $\varphi, \omega_{j}$ because $\operatorname{nip}\left(T \omega_{i}, T \omega_{j}\right)=0(i \neq j)$ is not necessarily materialized．That is，it is because the separation between patterns is not good unlike the following（ $1 \%$ ）．

Three expressions（8），（9），and（10）bring about respectively the effect
（1\％）$\omega_{i}$ can completely disassociate from $\omega_{j}$ about all the different $j$ from $i$
（2\％）If one certain degree $S M\left(\varphi, \omega_{i}\right)$ of similar about $\omega_{i}$ becomes maximum 1 ，the degree $S M\left(\varphi, \omega_{j}\right)$ of similar about $\omega_{j}$ with all different $j$ from $i$ will become the minimum value 0 and
（3\％）A pattern model $T \varphi$ can identify with the original pattern $\varphi$ from a viewpoint of the similarity
between patterns．
Furthermore，

$$
\begin{equation*}
\forall \varphi \in \Phi, \forall j \in\{1,2, \cdots, r\}, S M\left(a \cdot \varphi, \omega_{j}\right)=S M\left(\varphi, \omega_{j}\right) \tag{11}
\end{equation*}
$$

holds for the arbitrary positive constants $a$ ，because

$$
\begin{aligned}
& \forall \varphi, \forall j \in\{1,2, \cdots, r\}, S M\left(a \cdot \varphi, \omega_{j}\right) \\
& =\operatorname{SM}\left(T(a \cdot \varphi), \omega_{j}\right) \quad \because \quad \text { 式 }(10) \\
& =S M\left(T \varphi, \omega_{j}\right) \quad \because \quad \text { 式(5) } \\
& =S M\left(\varphi, \omega_{j}\right) \quad \because \quad \text { 式 }(10)
\end{aligned}
$$

．From a viewpoint of the similarity between patterns，expression（11）means
（4\％）a pattern multiplied by any positive constant is identified with the original pattern．
In addition，as shown in the structure form（A．15）of $S M$ ，it guarantees that the addition of the new triple $<\mathrm{A}[\mathrm{r}+1] \mathrm{R}[\mathrm{r}+1] \mathrm{B}[\mathrm{r}+1]>$ becomes easy．

## 2．5 The reasoning method

Consider triple $<\mathrm{A}[\mathrm{j}] \mathrm{R}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ of expression（3）．
First，the two symbol sequences $\mathrm{A}[\mathrm{j}]$ and $\mathrm{R}[\mathrm{j}]$ are respectively inputted into the text field 1 and the text field 2 in a screen．

At this time，the input pattern $\varphi$ with which noise may be included is obtained from the inputted symbol sequence $<\mathrm{A}[\mathrm{j}] \mathrm{R}[\mathrm{j}]>$ as a $2 q$－dimensional linear prediction coefficient vector．Since the associative output

$$
\begin{equation*}
\psi \equiv A(T \varphi) \tag{12}
\end{equation*}
$$

obtained from the paired associate operator A of expression（A．9）into which the noise removable model T $\varphi$ was inputted may not be completely in agreement with $\omega_{j}=T \eta_{j}$ ，the last output $\psi^{\prime}$ is determined by the following method of maximum similarity．The youngest number

$$
j=\underset{i=1 \sim r}{\arg } S M\left(\psi, \omega_{i}\right) \in\{1,2, \cdots, r\}
$$

is called for such that the similarity－measure $\operatorname{SM}\left(T \varphi, \omega_{i}\right)$ which gives the grade to which model $T \varphi$ resembles $\omega_{i}$ serves as the maximum ，and $\phi^{\prime}$ are set to be

$$
\begin{equation*}
\left(\phi^{\prime}=\right) \omega_{j} \tag{13}
\end{equation*}
$$

（the method of maximum similarity－measure）．Since the symbol sequence $<\mathrm{A}[\mathrm{j}] \mathrm{B}[\mathrm{j}]>$ can be found at this time， the symbol sequence $\mathrm{B}[\mathrm{j}]$ of the second half is displayed on the text field 3 in a screen．

The above process is repeated and its reasoning is made including the inheritance of properties．

## 2．6 The advantage of this semantic network

The pattern－model structural form $T$ ，the paired－associate operator A，and the similarity－measure function $S M$ used for the foregoing paragraph are original with this research which both has not appeared in an old pattern information processing theory except SS theory proposed by S．Suzuki．

It is shown by the following theorem 1 that the semantic network system can memorize and reason by pattern information processing as an action of effectively fundamental three functions
（1\＆）conclusion of modeling by $T$（The model of a model is the original model；expression（6））
（2\＆）interpolation nature of $A$（The designed pair is surely recollected correctly；expression（A．11））
and
（3\＆）orthogonality of $S M$（The similarity－measure between the designed pair and any different desiged pair is 0 ；expression（8））
exceeding the performance by symbol－sequence processing．
［Theorem 1］（The basic theorem of the semantic network reasoning by pattern－pair association）
About arbitrary $j(=1 \sim r), \mathrm{B}[\mathrm{j}]$ is surely reasoned from $<\mathrm{A}[\mathrm{j}] \mathrm{R}[\mathrm{j}]>$ on the semantic network by pattern－ pair association．
（Proof）It is $T \varphi=T \varphi_{j}$ when $<\mathrm{A}[\mathrm{j}] \mathrm{R}[\mathrm{j}]>$ is inputted．Since expression（A．11）is realized at this time，$\psi$ of expression（12）is $\psi=T \eta_{j}$ ．

Therefore，
$S M\left(T \varphi, \omega_{j}\right)=S M\left(\varphi, \omega_{j}\right)=1 \wedge\left[\forall i(\neq j), S M\left(T \varphi, \omega_{i}\right)=S M\left(\varphi, \omega_{i}\right)=0\right]$ follows from three expressions （7），（10），and（8）．

It turns out that expression（13）is materialized．Thus this proof finishes．

## 3．The semantic network system implemented in the Java language

The adopted semantic network，and the contents of operation and examination of the semantic network implemented in the Java language are explained during this chapter．

## 3．1 The adopted semantic network

The semantic network memorized is shown in Fig． 1 ［6］．


Fig． 1 The adopted semantic network to memorize
Notes：動物＝Animal，数＝Number，飛行 $=$ Flight，鳥 $=$ Bird，思考 $=$ Thinking，人間 $=$ Man，羽 $=$ Feather，太郎 $=$ Taro（individual name），花子 $=$ Hanako（individual name），顔 $=$ Face，

目＝Eye，カナリヤ＝Canary，黄＝Yellow，動作主＝Agent，受け取り主＝Receipt Lord， ピータ $=$ Peter（individual name），対象物 $=$ Subject

The semantic network of Fig． 1 is a semantic network shown in the figure 8.8 of Section 8．3（p．189）of reference［6］，and it is possible that the network consists of the following 17 triples（1\＃）～（17\＃）：
$(1 \#)<$ G1 is－a 贈与 $>$ ．
$(2 \#)<$ G1 受取主 花子 $>$ 。
$(3 \#)<\mathrm{G} 1$ 動作主 太郎 $>$ 。
$(4 \#)<\mathrm{G} 1$ 対象物 ピーター $>$ ．
$(5 \#)<$ ピーター is－a カナリヤ＞。
（6\＃）＜カナリヤ 色 黄＞。
（7\＃）＜カナリヤ is－a 鳥＞。
（8\＃）＜鳥 has－as－part 羽＞。
$(9 \#)<$ 鳥 can 飛行＞。
$(10 \#)<$ 鳥 is－a 動物＞．
$(11 \#)<$ 動物 年齢 数＞．
$(12 \#)<$ 太郎 is－a 人間＞．
$(13 \#)<$ 花子 is－a 人間＞．
$(14 \#)<$ 人間 can 思考＞．
$(15 \#)<$ 人間 is－a 動物＞。
$(16 \#)<$ 人間 has－as－part 顔＞．
（17\＃）＜顔 has－as－part 目＞．

To a sense sake incidentally，（1\＃）$\sim(4 \#)$ in Fig． 1 expresses the knowledge＂Taro presented Hanako for Peter．＂

## 3．2 Examination in a memory scene

3．2．1 A setup of two threshold value vectors $\vec{e}_{-}$and $\vec{e}_{+}$in the noise removable model－ construction operator $T$

Finally it was chosen with

$$
e_{+}[k]=0.05, e_{-}[k]=-e_{+}[k], k=1 \sim 2 q
$$

about two threshold value vectors $\vec{e}_{-}, \vec{e}_{+}$of expression（A．4）in the noise removable model－construction operator $T$ as a result of trial and error．

## 3．2．2 The check of an orthogonal system $\left\{\tilde{\varphi}_{k}\right\}_{k=1 \sim r}$

It was checked that the orthogonal nature of expression（A．7）of Appendix 2 is materialized in the form where inequality

$$
\left|\left(\tilde{\varphi}_{k}, \tilde{\varphi}_{\ell}\right)\right|<10^{-13} \text { if } k \neq \ell
$$

is filled．
In addition，it is $\left\|\tilde{\varphi}_{10}\right\| \fallingdotseq 0.1181 \leq\left\|\tilde{\varphi}_{j}\right\| \leq 2.3412 \doteqdot\left\|\tilde{\varphi}_{1}\right\|, j=1 \sim r$.

## 3．2．3 The check of the paired－associate mapping A being equipped with interpolation character

About expression（A．10）of appendix 2 ．，it was checked as a matter of fact that

$$
\left\|\left(\Delta A_{j}\right)\left(T \varphi_{k}\right)\right\|<10^{-12}, k=1 \sim j, j=1 \sim r-1
$$

is materialized．It was checked that expression（A．11）having the interpolation character is materialized in the form where inequality

$$
\left\|A_{k}\left(T \varphi_{j}\right)-T \eta_{j}\right\|<10^{-11}, j=1 \sim k, k=1 \sim r \quad \therefore \quad\left\|A\left(T \varphi_{j}\right)-T \eta_{j}\right\|<10^{-11}, j=1 \sim r
$$

is filled．
It was checked by the above that all the 17 triples（1\＃）$\sim(17 \#)$ are memorized correctly．

## 3．3 Examination in a recall scene

It was checked that theorem 1 holds good，i．e．，all of 17 triples（1\＃）$\sim(17 \#)$ are memorized correctly，and are correctly reasoned including the inheritance of properties．

For example，it sees and needs about $(9 \#)<$ 鳥 can 飛行＞．
We will examine whether $B$ is reasoned from $A$ ．
For example，the voice waveform「鳥」is shown in Fig． 2.


Fig． 2 A voice waveform of uttered＂bird＂．

In Table 1，the $q$－dimensional linear prediction vectors of two voice waveforms「鳥，can」are expressed as＂
$(\varphi)_{k-1}=$ The $k$－th component $C_{k}(k=1 \sim 16)$ of the $q$－dimensional linmear prediction vector of voice waveform「鳥」
and
$(\varphi)_{k+15}=$ The $k$－th component $C_{k}(k=1 \sim 16)$ of the $q$－dimensional linmear prediction vector of voice waveform「can」
＂．

Tab． 1 Input pattern $\varphi=\operatorname{col}\left((\varphi)_{0} \quad(\varphi)_{1} \quad \cdots \quad(\varphi)_{31}\right)$ ，its model
$T \varphi=\operatorname{col}\left((T \varphi)_{0} \quad(T \varphi)_{1} \quad \cdots \quad(T \varphi)_{31}\right)$ and its reasoning result $A(T \varphi)=\operatorname{col}\left((A(T \varphi))_{0} \quad(A(T \varphi))_{1} \quad \cdots \quad(A(T \varphi))_{31}\right)$.

|  | $\varphi$ | $T \varphi$ | $A(T \varphi)$ |
| :---: | :---: | :---: | :---: |
| 0 | 0.05083 | 0.0 | 0.0 |
| 1 | 0.53100 | 0.31509 | 0.31509 |
| 2 | －0．11527 | －0．06840 | －0．06840 |
| 3 | －0．97613 | －0．57922 | －0．57922 |
| 4 | 0.16785 | 0.09960 | 0.09960 |
| 5 | 1.12911 | 0.67000 | 0.67000 |
| 6 | －0．22541 | －0．13376 | －0．13376 |
| 7 | －1．61943 | －0．96094 | －0．96094 |
| 8 | 0.24076 | 0.14286 | 0.14286 |
| 9 | 1.68525 | 1.0 | 1.0 |
| 10 | －0．20807 | －0．12347 | －0．12347 |
| 11 | －1．50069 | －0．89048 | －0．89048 |
| 12 | 0.115332 | 0.09098 | 0.09098 |
| 13 | 1.35077 | 0.80152 | 0.80152 |
| 14 | －0．06786 | 0.0 | 0.0 |
| 15 | －0．60863 | －0．36115 | －0．36115 |
| 16 | 0.14085 | 0.08358 | 0.0 |
| 17 | 0.06207 | 0.0 | 0.0 |
| 18 | －0．20659 | －0．12259 | 0.0 |
| 19 | －0．06707 | 0.0 | 0.0 |
| 20 | 0.15243 | 0.09045 | 0.0 |
| 21 | 0.06560 | 0.0 | 0.06378 |
| 22 | －0．09211 | －0．05465 | －0．05083 |
| 23 | 0.00588 | 0.0 | 0.0 |
| 24 | 0.05430 | 0.0 | 0.0 |
| 25 | 0.09246 | 0.05486 | 0.14108 |
| 26 | 0.01978 | 0.0 | 0.0 |
| 27 | 0.00359 | 0.0 | 0.0 |
| 28 | －0．08681 | －0．05151 | 0.0 |
| 29 | －0．00777 | 0.0 | 0.09785 |
| 30 | 0.06393 | 0.0 | 0.0 |
| 31 | 0.01681 | 0.0 | 0.0 |

## Becoming clear from Tab． 1 is as follows．：

（1）Two facts

$$
(T \varphi)_{k}=0 \neq(\varphi)_{k}, k=0,14,17,19,21,23,24,26,27,29,30,31
$$

$(A(T \varphi))_{k}=0 \neq(T \varphi)_{k}, k=16,18,20,28$ shows that small noise is removed in the scene of asking for a model，and its recall．
（2）In order to deal with the memory matter $(9 \#)<$ 鳥 can 飛行＞，

$$
\max _{\ell=0 \sim 31}\left|(\varphi)_{\ell}\right|=1.68525>(A(T \varphi))_{k}=(T \varphi)_{k}=1.0
$$

is materialized when $k=9$ ．
（3）The equation $(A(T \varphi))_{k}=(T \varphi)_{k}, k=0 \sim 15$ is materialized in order to recollect＜鳥 can＞to＜鳥 飛行＞．
（4）Equation
$(A(T \varphi))_{k}=0=(T \varphi)_{k}, k=0,14,17,19,23,24,26,27,30,31$
shows that a zero component may be saved in recall．
（5）Equation

$$
(A(T \varphi))_{k} \neq 0=(T \varphi)_{k}, k=21,29
$$

shows that a non－zero component may be recollected from a zero component．
Formation of two equations

$$
\begin{aligned}
& S M\left(T \varphi, \omega_{9}\right)=S M\left(T \varphi, T \eta_{9}\right)=1.0 \\
& S M\left(T \varphi, \omega_{j}\right)=S M\left(T \varphi, T \eta_{j}\right)=0.0, j=1 \sim 8,10 \sim 17
\end{aligned}
$$

was confirmed．
The execution screen is shown in Fig． 3.


Fig． 3 One example that＂飛行＂was reasoned from＂鳥＂and＂can＂．

Time required for reasoning，a display，and transfer of a reasoning result by a sound etc．was 5812 mm second．

In addition，equation（A．12）of appendix 2．being materialized is checked，that is，although the 1 st to $k$－ th triples are correctly reasoned from $A_{k}(1 \leq k \leq m)$ ，the $k+1$－th to $r$－th triples are not reasoned correctly．

## 4．Conclusion

The old semantic network system has been made by symbol－sequence processing．In this paper，the original method of and memorizing the semantic network and performing reasoning there by pattern associative processing was proposed exceeding the performance by symbol－sequence processing．It was shown that it can be carried out using the model－construction operator $T$ ，the paired－associate mapping $A$ and the similarity－measure function $S M$ proposed by S．Suzuki．It was checked that many original advantages in the introduction are equipped in the semantic network system implemented in the Java language，when the contents of operation are examined．As the method of memorizing the triple＜人間 IS－A 動物＞for example，on a semantic network
（1）＂Three $q$－dimensional linear prediction coefficient vectors of three voice waveforms which are obtained by utterring respectively with 人間，IS－A，and 動物 were used＂
is originality of this research．Next，it is also originality of this research to have transposed
（2）triple＜人間 IS－A 動物 $>$ to if－then－rule if $<$ 人間 IS－A $>$ then＜人間 動物 $>$ ．
（3）＂System asks for the pattern model of the $2 q$－dimensional linear prediction coefficient vector of $<$ 人
間 IS－A＞and＜人間 動物＞，and a paired－associate operator which can recall the latter model from the former model is designed＂
was presented here．It is also originality of this research．These 3 originalities make it easy to design a semantic network，and also make it easy to add new triple to the designed semantic network ．

Moreover，for example，the following（4），（5）and（6）as a method of reasoning＜動物＞from＜人間 IS－A＞ was proposed：
（4）It asks for the model of the $2 q$－dimensional linear prediction coefficient vector of＜人間 IS－A $>$ ，and asks for the $2 q$－dimensional vector pair－associated from this model．
Then，
（5）the set of the $2 q$－dimensional linear prediction coefficient vector of $<1$ st term 3 rd term $>$ of triple containing＜人間 動物＞is prepared．
（6）What resembles this $2 q$－dimensional vector most is taken out from this set by applying the maximum method of similarity．
For this reason（6），this designed semantic network will be equipped with proof against noise．It will have predominancy to the semantic network designed by mere symbol processing．

The used pattern model form $T$ ，the pair－associate operator $A$ ，and the similarity－measure function $S M$ are original with this research which both have not appeared in an old pattern information processing theory except SS theory proposed by S．Suzuki．

Moreover，about noise－proof nature it came to have a performance beyond＂the function of memorizing and reasoning＂of the conventional semantic network system done by symbol－processing because（1\＆），（2\＆），
and（3\＆）of Section 2.6 act effectively fundamentally．
For example，the system is designed so that the inputted symbol sequence，a reasoning result，etc．may be told also with a sound．Furthermore，about triple＜太郎 IS－A人間＞，it is desirable to change to the system which gives directions with the sound＂太郎 IS－A＂and can answer with the sound＂人間＂．We think that basic technology for that was secured enough．

## References

［1］Shoichi Suzuki（鈴木昇一）：＂recognition engineering（認識工学）（Quantum theory of recognition）＂， Kashiwa Shobo publishing company（柏書房），Feb． 1975
［2］Shoichi Suzuki（鈴木昇一）：＂a new mathematical principle of a neural network（ニューラルネットの新数理）（A new－generalized error－backpropagation neural network with hidden layers that can learn an input－output relation from examples，based on a new mathematical point of view）＂，Modern Literature publishing company（近代文芸社），Sept． 1996
［3］Shoichi Suzuki（鈴木昇一）：＂mathematical general solution of a pattern recognition problem（パターン認識問題の数理的一般解決）（A newly－generalized means of solving pattern－recognition problems in the employ of categorical－membership knowledges，based on a new mathematical point of view）＂，Modern Literature publishing company，June 1997
［4］Shoichi Suzuki（鈴木昇一）：＂new deployment of a recognition intelligence information theory（認識知能情報論の新展開）（A newly－developed intelligence informatics about pattern－recognition that contains a potential theory of categorical－membership knowledges）＂，Modern Literature publishing company， Aug． 1998
［5］Shoichi Suzuki（鈴木昇一）：＂A general solution of＂a paired associate problem and its pseudoinverse problem＂by the help of a sequence of input－output pairs＂（＂入出力例の系列を用いた＂対連想問題• その擬逆問題＂の一般解＂），Information and Communication Studies of the Faculty of Information and Communication（Bunkyo University）（情報研究（文教大学•情報学部）），no．30，pp．81－137，Jan． 2004
［6］Ikuo Tahara（太原育夫）：＂The basic knowledge of artificial intelligence（computer science university lecture 20）（人工知能の基礎知識）＂，Kindai Kagaku Sha publishing company（近代科学社），Feb． 1999
［7］Shintani Toramatsu（新谷虎松）：＂a guide to intelligence programming by Java（Introduction to Knowledge Programming with Java）（Javaによる知能プログランミング入門）＂，Corona Publishing company（コロナ社），Oct． 2002
［8］Mori Kazuwo（守一雄），Takashi Tsuzuki（都築誉史），Tkasi Kusumi（楠見孝）：＂understanding of the heart by the simulation of brain－connectionist model and psychology－（コネクショニストモデルと心理学－脳のシミュレーションによる心の理解－）＂，Kitaoji Shobo publishing company（北大路書房）， Jun． 2001
［9］Teuvo Kohonen（T．コホネン）：＂Associative memory－for information engineering and psychology－ （Associative Memory A System－Theoretical approach）（システム論的連想記憶－情報工学•心理学の ために－）＂，translation by Kazuo Nakatani（中谷和夫），Saiensu－Sha publishing company（サイエンス社），Aug． 1980
［10］Manfred Spitzer（M．シュピッツアー）：＂the soul in a brain，i．e．，a network（map which a neural network
draws）（Geist im Netz）（脳 回路網の中の精神（ニューラルネットが描く地図））＂，translation by Toshiya Murai（村井俊哉）and Hirosi Yamagishi（山岸洋），Shin－yo－sha publishing company（新曜社）， Nov． 2001
［11］Shoichi Suzuki（鈴木昇一）：＂Generalization of canonical forms called corresponding models of patterns （パターンモデル（パターンの標準形）の一般形）＂，Information and Communication Studies of the Faculty of Information and Communication（Bunkyo University）（情報研究（文教大学•情報学部）），no．32，pp．169－218，Jan． 2005
［12］Yasuhisa Niimi（新美康永）：＂speech recognition（information science lecture E－19－3）（音声認識（情報科学講座E•19•3））＂，Kyoritsu shuppan publishing company（共立出版），and Oct． 1979
［13］Shoichi Suzuki（鈴木昇一）：＂A corresponding entropy model of a pattern（パターンのエントロピーモ デル）＂，The Transactions of the Institute of Electronics，Information and Communication Engineers（電子情報通信学会論文誌），vol．J77－D II ，no．11，pp．2220－2238，Nov． 1994

## Appendices

1．The adopted model－construction operator $T$ which brings about the canonical form of a pattern（a corresponding model of a pattern）
There are the following three kind（1\＃），（2\＃）and（3\＃）in the mapping $T$ which generates the pattern model $T \varphi$ used as instead of pattern $\varphi$ ：
（1\＃）Covariant forms under unitay coordinate transformations［13］
（2\＃）Invariant forms under unitay coordinate transformations［1］，［2］，［13］
（3\＃）What approximates the amplitude of a pattern with the limited or infinite values
The general form of such mapping $T$ is synthetically studied by reference［11］．
The mapping $T$ adopted in this semantic network system is the 3 rd thing（ $3 \#$ ），and is equipped with the effect of removing small noise．This mapping $T$ is explained below．

About the $2 q$－dimensional real number value vector

$$
\varphi=\operatorname{col}\left(\begin{array}{llllll}
a_{1} & a_{2} & \cdots & a_{k} & \cdots & a_{2 q} \tag{A.1}
\end{array}\right) \text { (column vector) }
$$

，the noise removable model

$$
T \varphi=\operatorname{col}\left(\begin{array}{llllll}
c_{1} & c_{2} & \cdots & c_{k} & \cdots & c_{2 q} \tag{A.2}
\end{array}\right)
$$

is defined as follows．Henceforth，$a_{k}, c_{k}(k=1 \sim 2 q)$ may be expressed as

$$
\begin{equation*}
(\varphi)_{k} \equiv a_{k},(T \varphi)_{k} \equiv c_{k}(k=1 \sim 2 q) \tag{A.3}
\end{equation*}
$$

Two threshold value vectors

$$
\begin{equation*}
\vec{e}_{-}=\operatorname{col}\left(e_{-}[1] \quad e_{-}[2] \quad \cdots \quad e_{-}[2 q]\right), \vec{e}_{+}=\operatorname{col}\left(e_{+}[1] \quad e_{+}[2] \quad \cdots \quad e_{+}[2 q]\right) \tag{A.4}
\end{equation*}
$$

which fill inequality

$$
-1 \leq e_{-}(i)<e_{+}(i) \leq+1, i=1 \sim 2 q
$$

are prepared．
Zero 0 －calculation rule

$$
\frac{a_{k}}{\max _{\ell=1 \sim 2 q}\left|a_{\ell}\right|}=0 \quad \text { if } \quad \max _{\ell=1 \sim 2 q}\left|a_{\ell}\right|=0
$$

is prepared．The $k$－th ingredient $c_{k}=(T \varphi)_{k}$ of $T \varphi$ is defined as

$$
\begin{aligned}
& c_{k}=(T \varphi)_{k} \equiv \\
& \left\{\begin{array}{l}
0 \text { if } \quad e_{-}[k]<\frac{a_{k}}{\max _{\ell=1 \sim 2 q}\left|a_{\ell}\right|}<e_{+}[k] \\
\frac{a_{k}}{\max _{\ell=1 \sim 2 q}\left|a_{\ell}\right|} \text { if } \frac{a_{k}}{\max _{\ell=1 \sim 2 q}\left|a_{\ell}\right|} \leq e_{-}[k] \text { or } \frac{a_{k}}{\max _{\ell=1 \sim 2 q}\left|a_{\ell}\right|} \geq e_{+}[k]
\end{array}\right.
\end{aligned}
$$

, and the obtained mapping $T$ is called a model-construction mapping.

## 2.The adopted paired associate mapping $A$ which brings about work of association

Although the paired-associate mapping $A$ on general abstract Hilbert space is studied by reference [5], it is discussed below by $2 q$-dimensional Euclidean space $R^{2 q}$ which is the special space.

The inner product $(\varphi, \eta)$ and the norm $\|\varphi\|$ are expressed in $R^{2 q}$ as

$$
\begin{equation*}
(\varphi, \eta)=\sum_{k=1}^{2 q}(\varphi)_{k} \cdot(\eta)_{k}, \quad\|\varphi\|=\sqrt{(\varphi, \varphi)} \tag{A.5}
\end{equation*}
$$

The mapping $A$ which recollects a pattern model $A(T \varphi)=T \eta$ from a pattern model $T \varphi$ is defined as follows.

First, let the system

$$
\begin{equation*}
T \varphi_{j}, j=1 \sim r \tag{A.6}
\end{equation*}
$$

of a pattern model $T \varphi_{j}$ be a linearly independent system. By the method
(1\&) $\tilde{\varphi}_{1}=T \varphi_{1}$
(2\&) By the application of

$$
\tilde{\varphi}_{j}=T \varphi_{j}-\sum_{k=1}^{j-1} \frac{\left(T \varphi_{j}, \tilde{\varphi}_{k}\right)}{\left(\tilde{\varphi}_{k}, \tilde{\varphi}_{k}\right)} \cdot \tilde{\varphi}_{k}, j=2,3, \cdots, r \quad \text { (Gram-Schmidt orthogonalization) }
$$

the orthogonality

$$
\begin{equation*}
\left(\tilde{\varphi}_{k}, \tilde{\varphi}_{\ell}\right)=0 \text { if } k \neq \ell \tag{A.7}
\end{equation*}
$$

holds. Thus an orthogonal system

$$
\begin{equation*}
\tilde{\varphi}_{j}, j=1 \sim m \tag{A.8}
\end{equation*}
$$

is obtained.Then, the sequence

$$
A_{1}, A_{2}, \cdots, A_{t}, \cdots, A_{r}
$$

of the linear operator $A_{t}$ will be defined as
$(1 \%) \quad A_{1}(T \varphi)=\frac{\left(T \varphi, \tilde{\varphi}_{1}\right)}{\left(\tilde{\varphi}_{1}, \tilde{\varphi}_{1}\right)} \cdot T \eta_{1}$
$(2 \%) \quad A_{j}(T \varphi)=A_{j-1}(T \varphi)+\left(\Delta A_{j-1}\right)(T \varphi), j=2,3, \cdots, r$
where

$$
\left(\Delta A_{j-1}\right)(T \varphi)=\frac{\left(T \varphi, \tilde{\varphi}_{j}\right)}{\left(\tilde{\varphi}_{j}, \tilde{\varphi}_{j}\right)} \cdot\left[T \eta_{j}-A_{j-1}\left(T \varphi_{j}\right)\right]
$$

. Finally, if the linear operator $A$ is defined as

$$
\begin{equation*}
A=A_{r} \tag{A.9}
\end{equation*}
$$

, the zero-property

$$
\begin{equation*}
\left(\Delta A_{j}\right)\left(T \varphi_{k}\right)=0, k=1 \sim j, j=1 \sim r-1 \tag{A.10}
\end{equation*}
$$

will be materialized．Therefore，the equation

$$
\begin{equation*}
A\left(T \varphi_{j}\right)=T \eta_{j}, j=1 \sim r \tag{A.11}
\end{equation*}
$$

which shows an interpolation property is realized．In addition，there is interpolation property

$$
\begin{equation*}
A_{t}\left(T \varphi_{j}\right)=T \eta_{j}, 1 \leq j \leq t \tag{A.12}
\end{equation*}
$$

also in $A_{t}$ in the middle of composition．Moreover，Expression

$$
A_{t}=A_{1}+\sum_{k=1}^{t-1}\left(\Delta A_{k}\right), 2 \leq t \leq r
$$

is materialized．

3．The adopted similarity－measure function $S M$ which brings about the work which measures the degree of similar between pattern models

Below，the similarity－measure function $S M$ in the Appendix 2 of reference［4］is explained．
Let all the $n$－dimensional real vectors that express pair＜concept1，concept2＞about triple ＜concept1，relation，concept2＞be

$$
\begin{equation*}
\omega_{j}, j=1 \sim r \tag{A.13}
\end{equation*}
$$

Similarity－measure function $\operatorname{SM}\left(\varphi, \omega_{j}\right)$ which gives the grade to which pattern $\varphi$ resembles Pattern $\omega_{j}$ is defined as follows．

First，normalized inner product $\operatorname{nip}(T \varphi, T \eta)$ is defined as

$$
\begin{align*}
& \operatorname{nip}(T \varphi, T \eta)= \\
& \left\{\begin{array}{l}
\frac{(T \varphi, T \eta)}{\|T \varphi\| \cdot\|T \eta\|} \text { if }\|T \varphi\| \cdot\|T \eta\| \neq 0 \\
0 \quad \text { if }\|T \varphi\| \cdot\|T \eta\|=0
\end{array}\right. \tag{A.14}
\end{align*}
$$

．Next，non－negative quantity $S\left(\varphi, \omega_{j}\right)$ is defined as

$$
S\left(\varphi, \omega_{j}\right)=-\frac{1}{2} \cdot \log _{e}\left[1-\left|\operatorname{nip}\left(T \varphi, T \omega_{j}\right)\right|^{2}\right]
$$

and the total

$$
S(\varphi)=\sum_{j=1}^{r} S\left(\varphi, \omega_{j}\right)
$$

is defined．At this time， $\operatorname{SM}\left(\varphi, \omega_{j}\right)$ is defined as

$$
\begin{align*}
& S M\left(\varphi, \omega_{j}\right)= \\
& \begin{cases}\frac{S\left(\varphi, \omega_{j}\right)}{S(\varphi)} & \text { if } \quad S(\varphi)>0 \\
\frac{1}{r} \quad \text { if } & S(\varphi)=0\end{cases} \tag{A.15}
\end{align*}
$$

## 4. How to ask for the linear prediction coefficients $C_{\ell}, \ell=1 \sim t$ of a voice waveform

The linear prediction coefficients of the voice waveform $f$ is obtained by solving the normal equation in a method of least squares[12]. Below, the procedure on $2 p+1$-dimensional Euclidean space $R^{2 p+1}$ is explained.

The voice waveform $f(x) \in R^{2 p+1}$ in the 1 -dimensional integer value section $\{x \mid x=-p \sim+p\}$ is approximated using the linear combination

$$
\sum_{\ell=1}^{t} C_{\ell} \cdot \phi_{\ell}
$$

of the past values

$$
\begin{equation*}
\phi_{\ell}(x)=f(x-\ell), \ell=1 \sim t \tag{A.16}
\end{equation*}
$$

of $t$ pieces. At this time,the linear combination coefficient $C_{\ell}(\varphi)=C_{\ell}$ approximated so that the squared norm

$$
\left\|f-\sum_{\ell=1}^{t} C_{\ell} \cdot \phi_{\ell}\right\|^{2}
$$

of the error

$$
f(x)-\sum_{\ell=1}^{t} C_{\ell} \cdot \phi_{\ell}(x), x \in\{x \mid x=-p \sim+p\}
$$

may be made into the minimum is called $\ell(=1 \sim t)$-th linear prediction coefficient of the voice waveform $f$.
.The inner product $(f, g)$ and $\|f\|$ in $R^{2 p+1}$ are defined as

$$
(f, g)=\sum_{x=-p}^{+p} f(x) \cdot g(x),\|f\|=\sqrt{(f, f)} .
$$

A method of least squares can be applied and it can calculate $C_{\ell}(f), \ell=1 \sim t$ as a solution of simultaneous linear equations

$$
\sum_{k=1}^{t} a_{\ell k} \cdot C_{k}(f)=b_{\ell}
$$

where

$$
a_{\ell k} \equiv\left(\psi_{k}, \phi_{\ell}\right), b_{\ell} \equiv\left(f, \psi_{\ell}\right) .
$$

However, it is assumed that the system of expression (A.16) is linearly independent.
. At this time, $f_{\perp} \in R^{2 p+1}$ exists such that

$$
\left(f_{\perp}, \phi_{\ell}\right)=0 \text { for arbitrary } \ell
$$

,and expression

$$
f=\sum_{\ell=1}^{t} C_{\ell}(f) \cdot \phi_{\ell}+f_{\perp}
$$

of the voice waveform $f \in R^{2 p+1}$ holds.
(Author "Shoichi Suzuki", subject matter "Memorization and Reasoning of Semantic Network Using Paired-Associate Mapping of Patterns", printing academic journal "Information and Communication Studies of The Faculty of Information and Communication (Bunkyo University)" ,no. 38, pp. 59-76 ,Sept. 2007, "contribution date" 9.5 , 2007)

