A Logical Approach To Negation

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0. Introduction

In discussions of negation in English there have been several approaches presented by linguists. First, one of the most comprehensive, syntactic analyses was attempted by Klima (1964). Following his article, the scope semantics of negation has been discussed by Jackendoff (1969, 1972), Lakoff (1970), Carden (1973), Ota (1980), and others.

The aim of this paper is to examine the role of the semantic interpretation of negative sentences containing some quantifiers (e. g. *some*, *all*, *many*, *both*, etc.). I shall give a logical approach to the negative sentences in English, by employing some notational conventions ($\forall$, $\exists$, $\sim$), which are usually used in the field of logic. To be more concrete, from this logical point of view I shall try to explain the ambiguities of negative sentences containing quantifiers, as in the following sentences.

(1) I couldn't read many of the books.

(2) John didn't solve all of the problems.

(3) All of the boys didn't go there.

(4) Both of the boys didn't go there.

I shall also try to explain the meaning differences, which are caused by the place where a negative element *not* appears in a sentence. Some examples are given:

(5) Not many of the arrows hit the target.

(6) Many of the arrows didn't hit the target.

(7) The target was not hit by many of the arrows.

(8) The arrows that did not hit the target were many.

1. Notational conventions

First, I shall show a key to some of the less familiar notational conventions employed in this paper:

∀ : universal quantifier ('All...')

∃ : existential quantifier ('There is some...')

～ : NEG, general negation markers

Universal means 'all', 'every', and a *universal quantifier* means that quantifiers include all elements in a set. The quantifiers of this kind include *all, every, each, both,* etc. On the other hand, *existential* means 'there is...', and an *existential quantifier* means that quantifiers include some elements in a set, but not all of the elements. Quantifiers of this kind include *many, much, a lot of, a few, a little,* numerals, etc. I employ the notation ∀ to stand for a universal quantifier, and ∃ to stand for an existential quantifier, and ～ to stand for negation. If, for example, the notation ～ semantically precedes Q(= quantifier) such as ～ Q, then the Q is influenced by negation (～). In other words, the Q is inside the scope of negation. To the contrary, if Q semantically precedes ～ as in Q～, the Q is not influenced by negation (～). It other words, the Q is out of the scope of negation. Let's consider the following examples.

(9) Not many of the problems were solved.

(10) Many of the problem weren't solved.

Sentence (9) will never be synonymous with (10). We can paraphrase (9), (10) as follows:

(11) There are not many (= few) of the problems that were solved.

(12) There are many of the problems that weren't solved.

As stated by Carden (1973), (9) corresponding to (11) can be called the Negative-Quantifier dialect, because the NEG is always construed with the quantifier *many*. On the other hand, (10) correspoding to (12) can be called the Negative-Verb dialect, because the NEG is always construed with the verb *solve*. Hence, from a logical point of view, we can describe the meaning difference between (9) and (10): logical elements Q(∀, ∃) and NEG(~) will give a reasonable account of the difference in meaning between (9) and (10). In the first case, the interpretation of (9) is given the representation, ~Q, because (9) is regarded as Negative-Quantifier dialect. In the second, the interpretation of (10) is given the representation, Q~, because (10) is interpreted as Negative-Verb dialect.

Logical operators are quite useful as one of the methods to account for those sentences which contain logical elements in them. Particulary in this paper I will only deal with the phenomena that occur in simple declarative sentences.

2. Negation and quantifiers

I shall adopt logical operators to describe negative sentences containing quantifiers. My analysis owes much to the literature of Ota (1977 a,b). I will discuss the circumstances under which something can be negated, and I will examine the meaning of negated items. I will also discuss a range of phenomena involving a negative element and its interaction with quantifiers. Then I will try to show a comprehensive description of

negative sentences containing quantifiers, by using logical operators (i. e. ∀, ∃, ～) stated in the previous section.

Before moving on to discuss a range of phenomena involving negation and its interaction with quantifiers, I will explain one of the most remarkable features of quantifiers. Among quantifiers some of them have nothing to do with the influence of negation, whereas some of them are much influenced by negation. Thus, (13) and (15) can be paraphrased as (14) and (16), respectively.

(13) I haven't solved many problems.

(14) There aren't many problems that I have solved.

(15) I haven't solved some problems.

(16) There are some problems that I haven't solved.

The quantifier *many* in (13) is inside the scope of negation. However, *some* in (15) is not within it. Generally speaking, it is said that, if a quantifier occurs to the left of *not*, it will be out of the scope of negation (with one class of exceptions, which I will return to). And if a quantifier (e. g. *many*, *much*, etc.) occurs to the right of *not*, it will normally be inside the scope of negation. But certain quantifiers (e. g. *some*, *several*, *a number of*, etc.) will always be non-negated as in (15), even if they occur to the right of *not*. From this fact Lasnik (1972) predicted that certain quantifiers like *some* cannot occur at all immediately after *not*, since in that position a quantifier can always be negated, whereas such quantifiers as *some* can always be non-negated. That prediction was attested to be correct by Lasnik, as demonstrated in example (17).

(17) *Not { some / seseral } of the problems were solved.

From these phenomena, Lasnik proposed that quantifiers could be di-

vided into two classes: [+ some] and [− some] quantifiers. Certain quantifiers like *some, several, a number of* have nothing to do with the effect of negation. He called such quantifiers [+ some] quantifiers. Whereas certain others like *many, much* are quite influenced by negation. He called such quantifiers [−some] quantifiers. He regarded those determiners (i. e. *some, several, a number of*, etc.) as markers of reference: they are inherently referential. Therefore they are in [+ some] quantifiers, and cannot be negated. That is why, although a negative element *not* can always make quantifiers non-referential, it fails to make the quantifiers in (17) non-referential.

We can say that (13) logically has the sense of ~Q since *many* is inside the scope of negation. On the other hand, (15) has the sense of Q~ because *some* is a [+some] quantifier and so it is outside the scope of negation. In a word, a quantifier is out of the scope of negation in case it is a [+some] quantifier: Q~ on one hand. And a quantifier is within the scope of negation in case it is a [−some] quantifier:~Q on the other.

Ota (1977a) presented the two points of view as to the classification

Table 1

	[−some]	[+some]
∀	all, every, both	each[2] (?)
∃	many, much, a dozen, a great many, a lot of, a great deal of, N (numeral)	some, a few, a little, a couple of, several, most, a number of, plenty of, at least N, Definite determiner+...[3] (e. g. *those many boys*)

of quantifiers : one is whether they are a universal quantifier (∀) or an existential quantifier (∃), the other is whether they are a [+some] or [−some] quantifier.

As Ota claimed, whether negative sentences containing quantifiers logically have the senses of ~Q or Q~ depends upon the entanglement of the following three factors: the first factor is whether Q is a [+some] or [−some] quantifier. The second one is the relative difference of the positions between a negative element and a quantifier in surface structure (i. e. whether or not a quantifier precedes *not*). The last one is whether Q is ∀ or ∃. I will add another factor to consider to the three factors stated above . The factor is about intonation. Consider sentence (18).

(18) I couldn't solve many of the problems.

(18a) I was able to solve few of the problems.

(18b) Many of the problems, I was unable to solve.

As pointed out by Lasnik (1972), from (18) we can get two readings in (18a, b), and each reading is not only determined by three factors stated above, but also by the intonational contour of (18). According to the intonation given to (18), the quantifier *many* can be either inside or outside the scope of negation; ~∃ or ∃~. If *many* is within the same intonational phrase as *not*, it will necessarily be negated. If *many* is not within the same intonational phrase as negation owing to a special intonation, it will be non-negated. Lasnik gave us the two possible intonation patterns, which may informally be illustrated below.

(18c) I couldn't solve many of the problems.

(18b) I couldn't solve many of the problems.

(18c) corresponds to (18a), and (18d) corresponds to (18b) semantically. Lasnik claimed that this fact indicates that intonational contours of a sentence is relevant to the determination of the scope of *not*, consequently the semantic rule follows the rules assigning stress and intonation contours. Furthermore he proposed *not* Scope Rule, as shown in (19).

(19) *Not* Scope Rule

(a) $\begin{matrix}\text{Quant} \\ [-\text{some}]\end{matrix}$ [+negated]/not X ____

(b) $\left\{\begin{matrix}\text{Adverbial} \\ \text{NP}\end{matrix}\right\}$ [+negated]/not ____

Lasnik assumes that on each cycle, the rules above will apply in the order given in (20).

(20) 1. syntactic transformations
2. stress and intonation contour rules
3. *not* scope rules

Not Scope Rule, say, applies to (21):

(21) I didn't accept many of John's results.

(21a) I accepted few of John's results.

(21b) I rejected many of John's results.

When (21) has a normal intonation, rule (19a) applies and (21) is synonymous with (21a):$\sim\exists$. When the special, marked intonation contour isolates *many*, from *not*, rule (19a) does not apply and *accept* is negated and the reading produced is roughly synonymous with (21b):$\exists\sim$.

What is more, rule (19a) explains one significant point about the scope of *not*. The scope of negation in English is costrained by the pos-

sessivized NP island; it is constrained when a possessive occurs between *not* and a quantifier. Lasnik first presented sentence (22), which has an ambiguous scope of negation.

(22) I couldn't understand the proofs of many of the theorems.

(22a) I could understand the proofs of few of the theorems.

(22b) There are many of the theorems whose proofs I couldn't understand.

If "many of the theorems" is inside the scope of negation, (22) will be synonymous with (22a). If the phrase is outside the scope of negation, (22b) will be the correct paraphrase. (22) has both structures of $\sim\exists$ corresponding to (22a) and $\exists\sim$ corresponding to (22b). But if the direct object NP semantically has a 'subject', that is, a possessive determiner, it is no longer possible for a quantifier in the object to be negated. Consider sentence (23).

(23) I couldn't understand Euclid's proofs of many of the theorems.

(23) has no reading analogous to (22a). It does have a reading corresponding to (22b) only, in which the quantifier is non–negated, that is, $\exists\sim$; (24) is a paraphrase of (23).

(24) There are many of the theorems whose proofs by Euclid I couldn't understand.

Taking these analyses into consideration, I could say that a [−some] quantifier must be isolated from *not* if a marked, abnormal intonation contour occurs; a [−some] quantifier must be outside the scope of negation. More generally, if there appears an intonational isolation between *not* and a [−some] quantifier, a [−some] quantifier will not be influenced by a negative element *not*. This generalization explains the fol-

lowing examples.

(25) Not many of the arrows hit the target.

(26) Many of the arrows didn't hit the target.

(27) The target was not hit by many of the arrows.

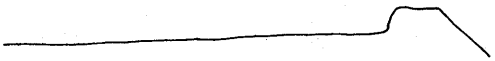
(28) The arrows that didn't hit the target were many.

The sense of (25) is logically $\sim\exists$. (27) has two readings: one is a normal, necessary reading and the other is a special, possible one. If (27) has the normal intonation contour of (27a), it is roughly equal to (25):$\sim\exists$.

(27a) The target was not hit by many of the arrows.

(27b) The target was not hit by many of the arrows.

This interpretation of $\sim\exists$ is a necessary one. On the other hand, if (27) has the marked or abnormal intonation contour of (27b), it is roughly synonymous with (26):$\exists\sim$. This interpretation of $\exists\sim$ is a possible one. This assumption of mine is confirmed by Jackendoff (1972). He assigned two intonation contours to sentence (27):



(27c) The target wasn't hit by many of the ARROWS.

(27d) The target wasn't HIT by MANY of the arrows.

Jackendoff assumes that (27c) and (27d) are synonymous with (25) and (26), respectively. Under Lasnik's analysis, (27) has a normal intonation contour, so a quantifier *many* should be inside the scope of negation:$\sim\exists$. On the other hand, (27d) has a marked intonation contour because the falling intonation occurs between *hit* and *many*, so that the intonation isolates *hit* from *many*. Therefore *many* should be outside the scope of negation:$\exists\sim$. This observation leads us to the conclusion that Lasnik's definition of *Not* Scope Rule is valid.

With regard to (27), Lasnik has suggested that intonational rules can optionally generate the contours of (27b), but that at the output the optionality is not completely free. By that, Lasnik has meant that the marked costruction can be generated but that if it ultimately serves no function, the sentence will be abnormal. In the case of (28), the meaning of (28) is only represented as $\exists\sim$, because the normal intonation contour isolates *not* from the [−some] quantifier *many*, and the marked intonation ultimately serves no function.

Ota (1977a) has proposed Table 2 to describe a relation between *not* and a quantifier. He takes two factors into account to decide whether a sense of negative sentences containing quantifiers is $\sim$Q or Q$\sim$: one facor is the relative difference of the position of *not* and a quantifier in surface structure. The other is an intonational isolation, which exists between *not* and a quantifier. The notation, ____ is used to show a position where a quantifier appears, and the notation, ... to show optioanl elements, and the notation, | means an existence of an intonational pause. The interpretations in parentheses, as in types of B, C, D, are less normal

ones.

Table 2

	[−some]		[+some]
	∀	∃	∃
A. not (very / too) ___	~∀	~∃	*
B. not ... ___	∀	~∃ (∃~)	∃~
C. not ∴ ___	~∀ (∀~)	∃~ (~∃)	∃~
D. ___ ... not	~∀ (∀~)	∃~ (~∃)	∃~
E. ___ ∴ not	∀~	∃~	∃~

From Table 2, note in particular that ∀ is more influenced by NEG(~) than ∃ is, and the effect of ~ is reduced as the circumstance change from type A to type E. On the basis of Table 2, I shall analyze the relationship between *not* and *both, all* in the next section.

3. Negation and *BOTH, ALL*

The aim of this section is to analyze negative sentences containing universal quantifiers *both, all*, which are located to the left of a negative element *not*. For example, example (29) is true to type D in Table 2, and

both and *all* are [− some] universal quantifiers. Therefore, on Ota's analysis, the relation between *both*, *all* and *not* is $\sim\forall$ or $\forall\sim$; (29a) corresponds to $\sim\forall$ and (29b) to $\forall\sim$.

(29) Both / All } of the boys didn't go there.

(29a) The boys didn't both go there.

Not all of the boys didn't go there.

(29b) None / Neither } of the boys went there.

Carden (1973) gave a full and detailed account of this matter. He distinguished three main dialects which can exist in such examples as (30). In one dialect, the negative is always construed with the quantifier, so that (30) means unambiguously (30a): $\sim\forall$. This he calls the Negative-Quantifier (NEG-Q) dialect. In the second, the negative is always construed with the verb, so that (30) unambiguously means (30b), i. e. "All the boys stayed" : $\forall\sim$.This he calls the Negative-Verb (NEG-V) dialect. In the third, (30) has both NEG-Q and NEG-V readings : $\sim\forall$ and $\forall\sim$. This he calls the ambiguous (AMB) dialect.

(30) All the boys didn't leave.

(30a) Not all the boys left.

(30b) All the boys (didn't leave).

See Table 3, given by Carden. He summarized the patterns of meaning and grammaticality for the three dialects defined above. In Table 3 he investigated the dialects defined above. He also investigated the dialect patterns for sentence (30).

Table 3

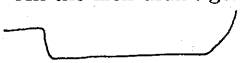
Dialect	Number of informants	Meaning
NEG–Q	16	$\sim\forall$
NEG–V	4	$\forall\sim$
AMB	20	$\sim\forall$, $\forall\sim$

From Table 3 we can say that $\sim\forall$ reading is more necessary interpretation than $\forall\sim$ reading.

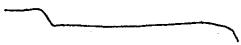
In this regard the contrast in meaning as shown in (30) is analized by Jackendoff (1972) from the viewpoint of intonation; whether the meaning of (30) is $\sim\forall$ or $\forall\sim$ is distinguished by a difference in the choice of pitch accent. He gave sentence (31) two intonation contours, as in (31a) and (31b). These are synonymous with (31c) and (31d), respectively.

(31) All the men didn't go.

(31a) All the men didn't go.



(31b) All the men didn't go.



(31c) Not all the men went.

(31d) None of the men went.

As stated in the previous section, Jackendoff's analysis leads to Lasnik's

idea of *not* scope rule. If sentence (29), (30) and (31) have a normal intonation contour, it gives us $\sim\forall$ reading. If they have a marked, special intonation contour, it gives us $\forall\sim$ reading. In this case the marked intonation contour necessarily contains a falling intonation. It follows from these facts that $\sim\forall$ reading is a necessary one, and $\forall\sim$ reading is a possible one.

4. Summary and conclusions

The purpose of this article has been to argue some ambiguities existing in negative sentences containing quantifiers in them. In order to analyze such sentences I have proposed a logical approach by introducing logical operators. Employing those logical operators I have analyzed the ambiguities occurring in negative sentences with quantifiers in them. This approach is based on the assumption that word order in surface structure is relevant to the scope interpretation of *not* and a quantifier, and the interpretation of scope of *not* and a quantifier must be accomlished at the level of derived structure. For this reason the semantic interpretations of negative sentences which are ambiguous in meaning, as shown in (18), (22), (29), etc., should be defined on the basis of their logical structure in derived structure.

NOTES

1. In general, all statements have logical structures. In other words, all statements consist of logical elements (e. g. *if*, *then*, *and*, *or*, *not*, *unless*, *some*, *all*, *every*, *any*, *it*, etc.) and others. For example, one may

say that statements (i) and (ii) have the same logical structure.

(i) *Every* microbe is an animal *or* a vegetable.

(ii) *Every* Genevan is a Calvinist *or* Catholic.

2. Some people claim that *each* belongs to [+ some]. This claim is, however, doubtful.

3. Definite determiner (e. g. *the, this/these, that/those, my, John's*) is one of the possessives.

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